AP Calculus BC Summer Readiness Packet

Differentiation Rules

d (f(g(x)) = f′(g(x)) g′(x) dx

d (u ± v) = du ± dv

d (uv) = u dv + v du

$d\left(\frac{u}{v}\right) = \frac{v \cdot du - u \cdot dv}{v^2}$

Basic Derivatives

d (k) = 0

$d(u^n) = nu^{n-1}\,du$

d sin(u) = cos(u) du

d cos(u) = –sin(u) du

$d\tan(u) = \sec^2(u)\,du$

d sec(u) = sec(u) tan(u) du

d csc(u) = –csc(u) cot(u) du

$d\cot(u) = -\csc^2(u)\,du$

$d\ln(u) = \frac{1}{u}du$

$d\,e^u = e^u\,du$

$d\,a^u = a^u \ln(a)\,du$

$d\log_a(u) = \frac{1}{u\ln(a)}du$

$d\sin^{-1}u = \frac{1}{\sqrt{1-u^2}}du$

$d\cos^{-1}u = \frac{-1}{\sqrt{1-u^2}}du$

$d\tan^{-1}u = \frac{1}{1+u^2}du$

$d\sec^{-1}u = \frac{1}{|u|\sqrt{u^2-1}}du$

$d\,u^v$ –> use logarithmic differentiation (not included in this packet)

Motion

$v(t) = x'(t)$

$a(t) = v'(t) = x''(t)$

$j(t) = a'(t) = v''(t) = x'''(t)$

$speed = |v(t)|$

Memorize the above formulas for derivatives.

Work the following problems on a separate piece of paper. This packet will serve as a study guide for your first quizzes of the semester.

Problems:

Find the first derivative.

1. $f(x) = 5x - 1$
2. $f(x) = x^2 + 3x - 4$
3. $y = x^{-2/5}$
4. $V(r) = \frac{4}{3}\pi r^3$
5. $f(x) = 6x^{-9}$
6. $f(x) = (16x)^3$
7. $g(x) = x^2 + \frac{1}{x^2}$
8. $y = \frac{x^2 + 4x + 3}{\sqrt{x}}$
9. $y = 3x + 2e^x$
10. $y = 4\pi^2$
11. $y = ax^2 + bx + c$
12. $y = x^2 e^x$
13. $y = \frac{x^2}{e^x}$
14. $y = (x^2 + x + 1)(x^2 + 2)$
15. $y = (1 + \sqrt{x})(x - x^3)$
16. $y = \frac{3x - 7}{x^2 + 5x - 4}$
17. $y = \frac{\sqrt{x} - 1}{\sqrt{x} + 1}$
18. $y = \frac{3x}{x^3 + 2x + 1}$
19. $y = x - 3\sin x$
20. $y = \sin x - \cos x$
21. $y = x^3 \cos x$
22. $y = \frac{\tan x}{x}$
23. $y = \csc x \cot x$
24. $y = \frac{\tan x - 1}{\sec x}$
25. $y = \tan x(\sin x + \cos x)$
26. $y = x \sin x \cos x$
27. $y = (x^3 + 4x)^7$
28. $y = \sqrt{x^2 - 7x}$
29. $y = \left(x - \frac{1}{x}\right)^{\frac{3}{2}}$
30. $y = e^{-2x}$
31. $y = (3x - 2)^{10}(5x^2 - x + 1)^{1/2}$
32. $y = \left(\frac{x - 6}{x + 7}\right)^3$
33. $y = 5^{-\frac{1}{x}}$
34. $y = \tan(\cos x)$
35. $y = \sin(\sin(\sin x))$
36. $x^2 + y^2 = 1$
37. $x^3 + x^2 y + 4y^2 = 6$
38. $\frac{y}{x - y} = x^2 + 1$
39. $\sqrt{xy} = 1 + x^2 y$
40. $4\cos x \sin y = 1$
41. $y = \sin^{-1}(x^2)$
42. $y = (1 + x^2)\arctan x$
43. $y = \arctan(\cos x)$
44. $f(x) = \ln(2 - x)$
45. $f(x) = \ln(\cos x)$
46. $y = \log_3(x^2 - 4)$
47. $y = e^x \ln x$
48. $y = (\ln(\tan x))^2$

Find the first and second derivatives.

1. $f(x) = x^5 + 6x^2 - 7x$
2. $f(x) = \cos 2x$
3. $f(x) = \sqrt{x^2 + 1}$
4. $f(x) = \frac{x}{1 - x}$
5. $f(x) = x^3 e^{5x}$

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Solve the following problems.

1. Find a parabola with equation $y = ax^2 + bx$ whose tangent line at (1,1) has equation $y = 3x - 2$.

2. Find an equation of the tangent line to the curve at the given point: $y = \frac{2x}{x+1}$, (1,1).

3. Find an equation of the tangent line to the curve at the given point: $y = \frac{e^x}{x}$, (1,*e*).

4. Find all points on the graph of the function $y = 2\sin x + \sin^2 x$ at which the tangent line is horizontal.

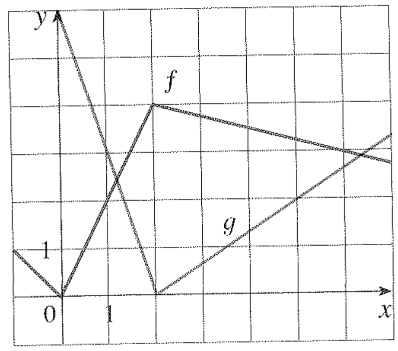
5. Suppose that $F(x) = f(g(x))$ and $g(3) = 6$, $g'(3) = 4$, $f'(3) = 2$, and $f'(6) = 7$. Find $F'(3)$.

6. A table of values for *f*, *g*, f', and g' is given.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	4	6
2	1	8	5	7
3	7	2	7	9

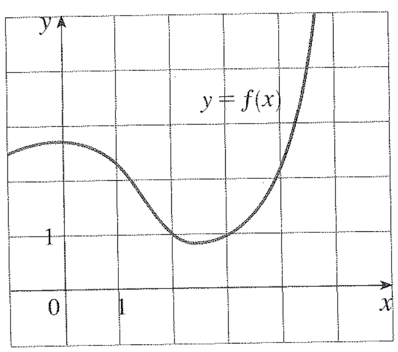
(a) If $h(x) = f(g(x))$, find $h'(1)$.
(b) If $H(x) = g(f(x))$, find $H'(1)$.
(c) If $F(x) = f(f(x))$, find $F'(2)$.
(d) If $G(x) = g(g(x))$, find $G'(3)$.

7. If *f* and *g* are the functions whose graphs are shown, let $u(x) = f(g(x))$, $v(x) = g(f(x))$, and $w(x) = g(g(x))$.



Find each derivative if it exists. If it does not exist, write DNE.
(a) $u'(1)$
(b) $v'(1)$
(c) $w'(1)$

8. If *f* is the function whose graph is shown, let $h(x) = f(f(x))$ and $g(x) = f(x^2)$. Use the graph of f to estimate each derivative.



(a) $h'(2)$
(b) $g'(2)$

9. Use the table to estimate the value of $g'(1)$, where $g(x) = f(f(x))$.

x	0.0	0.5	1.0	1.5	2.0	2.5
$f(x)$	1.7	1.8	2.0	2.4	3.1	4.4

10. If $x[f(x)]^3 + xf(x) = 6$ and $f(3) = 1$, find $f'(3)$.

11. Find an equation of the line tangent to $y = \ln(\ln x)$ at the point (*e*, 0).

12. If $f(x) = \frac{x}{\ln x}$, find $f'(e)$.

13. If $f(x) = (2-3x)^{-1/2}$, find $f(0)$, $f'(0)$, $f''(0)$, and $f'''(0)$.

14. Find $\frac{d^2y}{dx^2}$ by implicit differentiation: $x^3 + y^3 = 1$.

15. Find $\frac{d^2y}{dx^2}$ by implicit differentiation: $x^2 + xy + y^2 = 1$.

16. A particle's position is defined by $s(t) = t^3 - 12t^2 + 36t$, $t \geq 0$, where *s* is measured in meters and *t* is measured in seconds.
(a) Find the acceleration at time *t* and at time $t = 3$.
(b) When is the particle speeding up? When is it slowing down?

17. A mass attached to a vertical spring has position function given by $y = A\sin(\omega t)$, where *A* is the amplitude of its oscillations and ω is a constant.
(a) Find the velocity and acceleration as functions of time.
(b) Show that the acceleration is proportional to the displacement *y*.
(c) Show that the speed is a maximum when the acceleration is 0.